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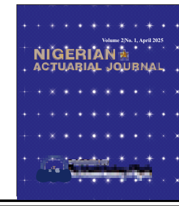

# ACTUARIAL UTILIZATION OF GM (1,2) MODEL IN SURVIVORSHIP ANALYSIS

**[*]A.T Chakfa, [2]G.M Ogungbenle, [1]F.M Abdulkarim, [1]A.A Muhammad, [1]S.D Bello, [3]S. Rawal**

**Abstract**

This study aims to provide an analytical utilization of the GM (1,2) model in survivorship estimation, with specific objectives as to determining the force of mortality, estimating the number and curve of survivors, and ascertaining the number and curve of deaths. The data used in this research was sourced from the National Center for Health Statistics 2022 (United States) provisional life table. Data preprocessing and cleansing were conducted using the R statistical tool to ensure accuracy and reliability before analysis. The methodology centers on an analytical estimation approach adopting the GM (1,2) model, which allows for effective solving of limited data peculiar of mortality studies. The models' major parameters (A,B,C) and auxiliary parameters $(k, s, g)$ were deduced following four equidistant backward simultaneous solution using survivorship values of $l_x$ which are $l_{20}, l_{40}, l_{60}, l_{80}$ as 98889, 95721,8665,54986 respectively to fully capture mortality trends across different age groups. The findings illustrate the GM (1,2) model's viability to estimate mortality dynamics. This study concludes that as age increases, the rate of deaths increases and exponentially increases as age goes older reaching omega $(\omega)$ where $l_x$ becomes infinitesimally zero.

**Keywords:** GM (1,2) model, Force of Mortality, Survivors, Curve of Death, GM (1,2) Auxiliary Parameters

## INTRODUCTION

GM (1,2) model been the Gomperz-Makeham law of mortality formulated as $\mu_x = A + BC^x$) by Makeham (1860) described the instantaneous rate of death with respect to age per period. Understanding mortality dynamics and survivorship composition is essential in demography, actuarial science, and public health planning. The analytical estimation of the force of mortality, survival curves, and death distributions enables policymakers and insurers to better anticipate population changes, allocate resources, and arrange social welfare programs within age range. Survivorship distribution used the auxiliary parametric values of GM (1,2) model, where death distribution can further be utilized $d_x = l_x - l_{x+1}$ as Traditional mortality modeling methods often require large, high-quality data, which may not always be available especially in developing countries with incomplete vital statistics. Gavrilov and Gavrilova (2001) explores aging and longevity by applying principles of reliability theory on Gompertz-Makeham model in mortality. Addressing these challenges, GM (1,2) and other related models aid these cases as opined by Ediev (2011) that hazard rate, birth cohorts differ substantially from the period life tables with respect to the distribution death. As $l_x$ is a continuous function, it can be differentiated and the ratio that the rate of decrease of $l_x$ at age $x$ bears to the value of $l_x$ (Adeyele. 2023a). The Makeham term serves as a fundamental component in mortality modeling, offering a constant additive hazard that accounts for background mortality factors usually unrelated to the aging process (Adeyele. 2023b; Patricio & Missov. 2024). Also, Castellares, Patricio and Lemonte (2022) observed that the results of GM (1,2) model will help determines the policies in insurance, established actuarial tables and growth models.



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The GM (1,2) model has gained attention for its ability to integrate multiple parameters and produce reliable predictions with limited data requirements. It extends the basic GM (0,2) model (Gompertz formula) by incorporating additional parameter to explain factors, improving forecast accuracy in complex systems such as mortality estimation (Adeyele, 2011; Ediev. 2011). Recent studies have successfully applied GM (1,2) models to estimate demographic indicators, discovering their potential to felicitating the traditional methods in mortality analysis. Ogungbenle, Sirisena, Ukwu and Adeyele. (2024) developed and implemented model for age dependent mortality rates in functional forms which presented critical modelling solution to life insurance and annuity firms using the generalized Makeham's framework. This research aims to analytically utilize the GM (1,2) model to estimate the force of mortality, determine the number of survivors, and ascertain the number of deaths, thereby providing an effective approach to survivorship estimation that will be impactful in life and health insurance.

**MATERIALS AND METHODS**

The GM(1,2) model in the force of mortality studies posed by Makeham was stated as

$$\mu_x = A + BC^x) \qquad (1)$$

$$\int_0^x \mu_x dt = \int_0^x \left(A + BC^x\right)dt \qquad (2)$$

$$= A_x + \frac{BC^x}{\log_e C} - \frac{B}{\log_e C} \qquad (3)$$

$$= -\log_e s^x - \log_e g^{c^x - 1} \qquad (4)$$

Since $A = -\log_e s^x$

$$l_x = l_0 e^{\log_e s^x + \log_e g^{c^x - 1}} \qquad (5)$$

But $k = \dfrac{l_0}{g}$

$$\text{Hence, } l_x = ks^x g^{c^x} \qquad (6)$$

From here, C can be determine and *k, s, g* in succession. This can be done using the values of $x$ and $l_x$ as 20,40,60,80 and 98889, 9571, 86665, 54986 respectively. $l_x$ represents the number of people living to age $x$

This implies,

$$\ln l_{20} = \ln k + 20 \ln s + c^{20} \ln g \qquad (8)$$

$$\ln l_{40} = \ln k + 40 \ln s + c^{40} \ln g \qquad (9)$$

$$\ln l_{40} = \ln k + 40 \ln s + c^{40} \ln g \qquad (10)$$

$$\ln l_{80} = \ln k + 80 \ln s + c^{80} \ln g \qquad (11)$$

$$\ln 98889 = \ln k + 20 \ln s + c^{20} \ln g \qquad (12)$$

$$\ln 95721 = \ln k + 40 \ln s + c^{40} \ln g \qquad (13)$$

$$\ln 86665 = \ln k + 60 \ln s + c^{60} \ln g \qquad (14)$$

$$\ln 54986 = \ln k + 80 \ln s + c^{80} \ln g \qquad (15)$$

$$(13) - (12), (14) - (13), (15) - (14)$$

$$\ln 95721 - \ln 98889 = 20 \ln s + c^{20}(c^{20} - 1) \ln g \qquad (16)$$

$$\ln 86665 - \ln 95721 = 20 \ln s + c^{40}(c^{20} - 1) \ln g \qquad (17)$$

$$\ln 54986 - \ln 86665 = 20 \ln s + c^{60}\left(c^{20} - 1\right) \ln g \qquad (18)$$

$$\ln\left(\frac{95721}{98889}\right) = 20 \ln s + c^{20}\left(c^{20} - 1\right) \ln g \qquad (19)$$

$$\ln\left(\frac{86665}{95721}\right) = 20 \ln s + c^{40}\left(c^{20} - 1\right) \ln g \qquad (20)$$

$$\ln\left(\frac{54986}{86665}\right) = 20 \ln s + c^{60}\left(c^{20} - 1\right) \ln g \qquad (21)$$

$$(21) - (19), (21) - (20)$$

$$\ln\left(\frac{86665}{95721} - \frac{95721}{98889}\right) = c^{20}\left(c^{20} - 1\right)^2 \ln g \qquad (22)$$

$$\ln\left(\frac{54986}{86665} - \frac{86667}{95721}\right) = c^{40}\left(c^{20} - 1\right)^2 \ln g \qquad (23)$$

$$\ln\left(\frac{86665}{95721} \times \frac{98889}{95721}\right) = c^{20}\left(c^{20} - 1\right)^2 \ln g \qquad (24)$$

$$\ln\left(\frac{54986}{86665} \times \frac{95721}{86665}\right) = c^{40}\left(c^{20} - 1\right)^2 \ln g \qquad (25)$$

$$-0.0668279987 = c^{20}\left(c^{20} - 1\right)^2 \ln g \qquad (26)$$

$$-0.3556346612 = c^{40}\left(c^{20} - 1\right)^2 \ln g \qquad (27)$$

$$(27) \div (26)$$

$$C^{20} = 5.32175423535 \qquad (28)$$

C will result to

$$C = 1.0871832203 \qquad (29)$$

$$\ln g = -0.00067232455 \qquad (30)$$

$$\ln g = -0.00067232455 \qquad (31)$$

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$$g = e^{-0.00067232455} \quad (32)$$

$$g = 0.99932790141 \quad (33)$$

From (12), solve for $s$

$$-0.03256029886 = 20 \ln s - 0.0156300 \quad (34)$$

$$\ln s = -0.00085486477 \quad (35)$$

$$s = e^{-0.00085486477} \quad (36)$$

$$s = 0.9991450052 \quad (37)$$

but $k = \frac{l_0}{g}$

$$\rightarrow k = \frac{100000}{0.99932790141} \quad (38)$$

$$k = 100067.5554655996$$

With the values of *k, s, g* been estimated, the GM (1,2) parameters will be determined

$$A = -\ln s \quad (39)$$

$$\rightarrow A = -(-0.00085486477)$$

$$A = 0.0008548677$$

$$B = -(\ln g)(\ln c) \quad (40)$$

$$\rightarrow B = -(-0.00067232455)(0.08359014987)$$

$$B = 0.00005619971$$

**Table 1: Summary Statistics of Variables**

| Parameters | Values |
|---|---|
| A | 0.000854865 |
| B | 5.52982E-05 |
| C | 1.08718322 |
| **Auxiliary Parameters** | **Values** |
| K | 100067.5555 |
| S | 0.999145005 |
| G | 0.999327901 |

**Source: Researchers' Computation**

**Table 2: GM (1,2) Force of Mortality**

| x | A | B | C | $C^x$ | $\mu_x$ |
|---|---|---|---|---|---|
| **0** | 0.0008549 | 5.61997E-05 | 1.08718322 | 1 | 0.000911064 |
| 1 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.08718322 | 0.000915964 |
| 2 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.181967355 | 0.000921291 |
| 3 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.285015075 | 0.000927082 |
| 4 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.397046827 | 0.000933378 |
| 5 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.518845868 | 0.000940223 |
| 6 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.651263742 | 0.000947665 |
| 7 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.795226233 | 0.000955756 |
| 8 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1.951739837 | 0.000964552 |
| 9 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2.121898801 | 0.000974115 |
| 10 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2.306892772 | 0.000984511 |
| 11 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2.508015113 | 0.000995814 |
| 12 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2.726671947 | 0.001008103 |
| 13 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2.964391988 | 0.001021463 |
| 14 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3.222837228 | 0.001035987 |
| 15 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3.503814556 | 0.001051778 |

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| $x$ | A | B | C | $C^x$ | $\mu_x$ |
|---|---|---|---|---|---|
| 16 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3.809288392 | 0.001068946 |
| 17 | 0.000854865 | 5.61997E-05 | 1.08718322 | 4.141394421 | 0.00108761 |
| 18 | 0.000854865 | 5.61997E-05 | 1.08718322 | 4.502454523 | 0.001107901 |
| 19 | 0.000854865 | 5.61997E-05 | 1.08718322 | 4.894993008 | 0.001129962 |
| 20 | 0.000854865 | 5.61997E-05 | 1.08718322 | 5.321754261 | 0.001153946 |
| 21 | 0.000854865 | 5.61997E-05 | 1.08718322 | 5.785721936 | 0.001180021 |
| 22 | 0.000854865 | 5.61997E-05 | 1.08718322 | 6.290139806 | 0.001208369 |
| 23 | 0.000854865 | 5.61997E-05 | 1.08718322 | 6.83853445 | 0.001239188 |
| 24 | 0.000854865 | 5.61997E-05 | 1.08718322 | 7.434739906 | 0.001272695 |
| 25 | 0.000854865 | 5.61997E-05 | 1.08718322 | 8.082924473 | 0.001309123 |
| 26 | 0.000854865 | 5.61997E-05 | 1.08718322 | 8.787619858 | 0.001348726 |
| 27 | 0.000854865 | 5.61997E-05 | 1.08718322 | 9.553752855 | 0.001391783 |
| 28 | 0.000854865 | 5.61997E-05 | 1.08718322 | 10.3866798 | 0.001438593 |
| 29 | 0.000854865 | 5.61997E-05 | 1.08718322 | 11.29222399 | 0.001489484 |
| 30 | 0.000854865 | 5.61997E-05 | 1.08718322 | 12.27671644 | 0.001544813 |
| 31 | 0.000854865 | 5.61997E-05 | 1.08718322 | 13.34704011 | 0.001604965 |
| 32 | 0.000854865 | 5.61997E-05 | 1.08718322 | 14.51067805 | 0.001670361 |
| 33 | 0.000854865 | 5.61997E-05 | 1.08718322 | 15.77576569 | 0.001741458 |
| 34 | 0.000854865 | 5.61997E-05 | 1.08718322 | 17.15114775 | 0.001818754 |
| 35 | 0.000854865 | 5.61997E-05 | 1.08718322 | 18.64644004 | 0.001902789 |
| 36 | 0.000854865 | 5.61997E-05 | 1.08718322 | 20.27209673 | 0.001994151 |
| 37 | 0.000854865 | 5.61997E-05 | 1.08718322 | 22.03948341 | 0.002093477 |
| 38 | 0.000854865 | 5.61997E-05 | 1.08718322 | 23.96095654 | 0.002201464 |
| 39 | 0.000854865 | 5.61997E-05 | 1.08718322 | 26.0499499 | 0.002318864 |
| 40 | 0.000854865 | 5.61997E-05 | 1.08718322 | 28.32106842 | 0.002446501 |
| 41 | 0.000854865 | 5.61997E-05 | 1.08718322 | 30.79019037 | 0.002585265 |
| 42 | 0.000854865 | 5.61997E-05 | 1.08718322 | 33.47457832 | 0.002736126 |
| 43 | 0.000854865 | 5.61997E-05 | 1.08718322 | 36.39299985 | 0.002900141 |
| 44 | 0.000854865 | 5.61997E-05 | 1.08718322 | 39.56585877 | 0.003078455 |
| 45 | 0.000854865 | 5.61997E-05 | 1.08718322 | 43.01533776 | 0.003272314 |
| 46 | 0.000854865 | 5.61997E-05 | 1.08718322 | 46.76555342 | 0.003483075 |
| 47 | 0.000854865 | 5.61997E-05 | 1.08718322 | 50.84272497 | 0.003712211 |
| 48 | 0.000854865 | 5.61997E-05 | 1.08718322 | 55.27535746 | 0.003961324 |
| 49 | 0.000854865 | 5.61997E-05 | 1.08718322 | 60.09444113 | 0.004232155 |
| 50 | 0.000854865 | 5.61997E-05 | 1.08718322 | 65.33366803 | 0.004526598 |
| 51 | 0.000854865 | 5.61997E-05 | 1.08718322 | 71.0296676 | 0.004846711 |
| 52 | 0.000854865 | 5.61997E-05 | 1.08718322 | 77.22226276 | 0.005194734 |
| 53 | 0.000854865 | 5.61997E-05 | 1.08718322 | 83.95474831 | 0.005573097 |
| 54 | 0.000854865 | 5.61997E-05 | 1.08718322 | 91.27419362 | 0.005984448 |
| 55 | 0.000854865 | 5.61997E-05 | 1.08718322 | 99.23177175 | 0.006431662 |
| 56 | 0.000854865 | 5.61997E-05 | 1.08718322 | 107.8831172 | 0.006917865 |
| 57 | 0.000854865 | 5.61997E-05 | 1.08718322 | 117.2887147 | 0.007446457 |
| 58 | 0.000854865 | 5.61997E-05 | 1.08718322 | 127.5143226 | 0.008021133 |

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| $x$ | A | B | C | $C^x$ | $\mu_x$ |
|---|---|---|---|---|---|
| 59 | 0.000854865 | 5.61997E-05 | 1.08718322 | 138.6314319 | 0.008645911 |
| 60 | 0.000854865 | 5.61997E-05 | 1.08718322 | 150.7177665 | 0.00932516 |
| 61 | 0.000854865 | 5.61997E-05 | 1.08718322 | 163.8578268 | 0.010063627 |
| 62 | 0.000854865 | 5.61997E-05 | 1.08718322 | 178.1434798 | 0.010866477 |
| 63 | 0.000854865 | 5.61997E-05 | 1.08718322 | 193.674602 | 0.011739321 |
| 64 | 0.000854865 | 5.61997E-05 | 1.08718322 | 210.5597775 | 0.012688263 |
| 65 | 0.000854865 | 5.61997E-05 | 1.08718322 | 228.917057 | 0.013719937 |
| 66 | 0.000854865 | 5.61997E-05 | 1.08718322 | 248.8747832 | 0.014841555 |
| 67 | 0.000854865 | 5.61997E-05 | 1.08718322 | 270.5724883 | 0.01606096 |
| 68 | 0.000854865 | 5.61997E-05 | 1.08718322 | 294.1618691 | 0.017386677 |
| 69 | 0.000854865 | 5.61997E-05 | 1.08718322 | 319.8078482 | 0.018827973 |
| 70 | 0.000854865 | 5.61997E-05 | 1.08718322 | 347.6897262 | 0.020394927 |
| 71 | 0.000854865 | 5.61997E-05 | 1.08718322 | 378.0024362 | 0.022098492 |
| 72 | 0.000854865 | 5.61997E-05 | 1.08718322 | 410.9579059 | 0.02395058 |
| 73 | 0.000854865 | 5.61997E-05 | 1.08718322 | 446.7865396 | 0.025964139 |
| 74 | 0.000854865 | 5.61997E-05 | 1.08718322 | 485.7388289 | 0.028153246 |
| 75 | 0.000854865 | 5.61997E-05 | 1.08718322 | 528.0871042 | 0.030533207 |
| 76 | 0.000854865 | 5.61997E-05 | 1.08718322 | 574.1274385 | 0.03312066 |
| 77 | 0.000854865 | 5.61997E-05 | 1.08718322 | 624.1817175 | 0.035933696 |
| 78 | 0.000854865 | 5.61997E-05 | 1.08718322 | 678.5998897 | 0.038991982 |
| 79 | 0.000854865 | 5.61997E-05 | 1.08718322 | 737.7624134 | 0.042316898 |
| 80 | 0.000854865 | 5.61997E-05 | 1.08718322 | 802.0829164 | 0.045931692 |
| 81 | 0.000854865 | 5.61997E-05 | 1.08718322 | 872.011088 | 0.049861635 |
| 82 | 0.000854865 | 5.61997E-05 | 1.08718322 | 948.0358227 | 0.054134203 |
| 83 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1030.688639 | 0.058779267 |
| 84 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1120.547393 | 0.063829303 |
| 85 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1218.240324 | 0.069319618 |
| 86 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1324.450438 | 0.075288595 |
| 87 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1439.920292 | 0.081777968 |
| 88 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1565.457181 | 0.088833104 |
| 89 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1701.938779 | 0.096503331 |
| 90 | 0.000854865 | 5.61997E-05 | 1.08718322 | 1850.319282 | 0.104842272 |
| 91 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2011.636076 | 0.113908229 |
| 92 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2187.016987 | 0.123764585 |
| 93 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2377.688171 | 0.13448025 |
| 94 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2584.982682 | 0.146130142 |
| 95 | 0.000854865 | 5.61997E-05 | 1.08718322 | 2810.349797 | 0.158795708 |
| 96 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3055.365143 | 0.1725655 |
| 97 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3321.741715 | 0.187535786 |
| 98 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3611.341855 | 0.20381123 |
| 99 | 0.000854865 | 5.61997E-05 | 1.08718322 | 3926.190267 | 0.221505619 |
| 100 | 0.000854865 | 5.61997E-05 | 1.08718322 | 4268.488178 | 0.240742663 |

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| $x$ | A | B | C | $C^x$ | $\mu_x$ |
|---|---|---|---|---|---|
| 101 | 0.000854865 | 5.61997E-05 | 1.08718322 | 4640.628723 | 0.261656853 |
| 102 | 0.000854865 | 5.61997E-05 | 1.08718322 | 5045.21368 | 0.28439441 |
| 103 | 0.000854865 | 5.61997E-05 | 1.08718322 | 5485.071655 | 0.309114301 |
| 104 | 0.000854865 | 5.61997E-05 | 1.08718322 | 5963.277866 | 0.335989351 |
| 105 | 0.000854865 | 5.61997E-05 | 1.08718322 | 6483.175634 | 0.365207455 |
| 106 | 0.000854865 | 5.61997E-05 | 1.08718322 | 7048.399763 | 0.396972887 |
| 107 | 0.000854865 | 5.61997E-05 | 1.08718322 | 7662.901953 | 0.431507732 |
| 108 | 0.000854865 | 5.61997E-05 | 1.08718322 | 8330.978422 | 0.469053436 |
| 109 | 0.000854865 | 5.61997E-05 | 1.08718322 | 9057.299949 | 0.509872495 |
| 110 | 0.000854865 | 5.61997E-05 | 1.08718322 | 9846.944525 | 0.554250291 |
| 111 | 0.000854865 | 5.61997E-05 | 1.08718322 | 10705.43286 | 0.602497087 |
| 112 | 0.000854865 | 5.61997E-05 | 1.08718322 | 11638.76697 | 0.654950193 |
| 113 | 0.000854865 | 5.61997E-05 | 1.08718322 | 12653.47216 | 0.71197633 |
| 114 | 0.000854865 | 5.61997E-05 | 1.08718322 | 13756.64261 | 0.77397419 |
| 115 | 0.000854865 | 5.61997E-05 | 1.08718322 | 14955.99101 | 0.841377222 |
| 116 | 0.000854865 | 5.61997E-05 | 1.08718322 | 16259.90247 | 0.914656668 |
| 117 | 0.000854865 | 5.61997E-05 | 1.08718322 | 17677.49313 | 0.994324852 |
| 118 | 0.000854865 | 5.61997E-05 | 1.08718322 | 19218.6739 | 1.080938765 |
| 119 | 0.000854865 | 5.61997E-05 | 1.08718322 | 20894.21979 | 1.175103957 |
| 120 | 0.000854865 | 5.61997E-05 | 1.08718322 | 22715.84515 | 1.277478775 |

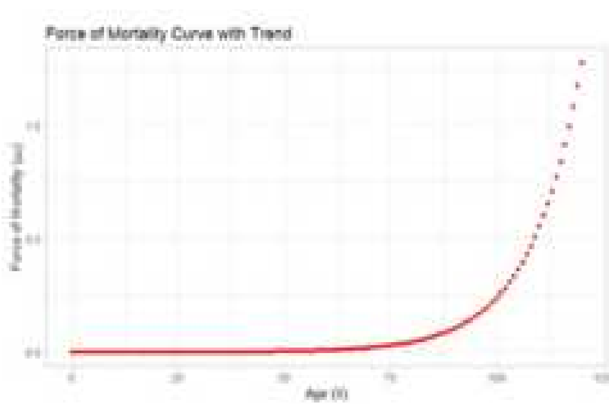


Table 2 presents a detailed numerical output of the GM (1,2) model regarding force of mortality, representing the age-independent and age-dependent variables. The force of mortality, denoted from this study increases exponentially with age, a key attribute of the Gompertz component which models the biological aging process. Initially, the mortality rate. remains nearly constant, with very slow increment during early ages (e.g., from 0.000911 at age 0 to pproximately 0.001545 by age 30). However, the xponential nature becomes rapid as age increases, ompatible with empirical mortality patterns bserved in actuarial and demographic studies Makeham, 1860; Gompertz, 1825). The parameters A, B, and C represent the baseline mortality, the initial evel of the Gompertzian function, and the rate of xponential increase in mortality with age, espectively. The graphical presentation of the nortality curve shows a flat segment in early life, ollowed by a sharp exponential increase from midlife orwards, ultimately reaching high values in old age. The exponential curvature visualized real-world observations where mortality risk rises gradually in adulthood but escalates significantly beyond age 60 (Oeppen & Vaupel. 2002). By age 90, the force of mortality approaches 0.1048, and it exceeds 1.0 at around age 118, indicating a very high death rate in late old age, a point at which the human lifespan attained its limits (Omega) just as in the study by (Gavrilov, Leonid, Gavrilova & Natalia 2011). The GM (1,2) modeling thus proves it usefulness for

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predicting mortality, evaluating life insurance liabilities, and assessing longevity risk in pension schemes and its ability to integrate both constant (accidental) and age-increasing risks. (Ogungbenle, Sogunro & Ogungbenle 2024).

**Table 3: Survivors Table**

| $x$ | K | $s^x$ | $g^{c(x)}$ | $l_x$ |
|---|---|---|---|---|
| 0 | 100067.5555 | 1 | 0.999327901 | 100000.3002 |
| 1 | 100067.5555 | 0.999145005 | 0.999269327 | 99908.94409 |
| 2 | 100067.5555 | 0.998290741 | 0.99920565 | 99817.16134 |
| 3 | 100067.5555 | 0.997437208 | 0.999136426 | 99724.90886 |
| 4 | 100067.5555 | 0.996584404 | 0.999061172 | 99632.13983 |
| 5 | 100067.5555 | 0.99573233 | 0.998979364 | 99538.80346 |
| 6 | 100067.5555 | 0.994880984 | 0.998890431 | 99444.84455 |
| 7 | 100067.5555 | 0.994030366 | 0.998793753 | 99350.2032 |
| 8 | 100067.5555 | 0.993180475 | 0.998688658 | 99254.81436 |
| 9 | 100067.5555 | 0.992331311 | 0.998574412 | 99158.60742 |
| 10 | 100067.5555 | 0.991482873 | 0.998450222 | 99061.50568 |
| 11 | 100067.5555 | 0.99063516 | 0.998315221 | 98963.42592 |
| 12 | 100067.5555 | 0.989788172 | 0.998168471 | 98864.27776 |
| 13 | 100067.5555 | 0.988941908 | 0.998008951 | 98763.96311 |
| 14 | 100067.5555 | 0.988096368 | 0.997835553 | 98662.37546 |
| 15 | 100067.5555 | 0.987251551 | 0.997647072 | 98559.39923 |
| 16 | 100067.5555 | 0.986407456 | 0.997442199 | 98454.90895 |
| 17 | 100067.5555 | 0.985564083 | 0.997219512 | 98348.76845 |
| 18 | 100067.5555 | 0.984721431 | 0.996977466 | 98240.82995 |
| 19 | 100067.5555 | 0.983879499 | 0.996714386 | 98130.93308 |
| 20 | 100067.5555 | 0.983038287 | 0.996428447 | 98018.90382 |
| 21 | 100067.5555 | 0.982197795 | 0.996117673 | 97904.55336 |
| 22 | 100067.5555 | 0.98135802 | 0.995779914 | 97787.67688 |
| 23 | 100067.5555 | 0.980518964 | 0.995412839 | 97668.05215 |
| 24 | 100067.5555 | 0.979680626 | 0.995013914 | 97545.43817 |
| 25 | 100067.5555 | 0.978843004 | 0.994580391 | 97419.57357 |
| 26 | 100067.5555 | 0.978006098 | 0.994109286 | 97290.17489 |
| 27 | 100067.5555 | 0.977169908 | 0.993597362 | 97156.93483 |
| 28 | 100067.5555 | 0.976334433 | 0.993041106 | 97019.52028 |
| 29 | 100067.5555 | 0.975499672 | 0.992436707 | 96877.57018 |
| 30 | 100067.5555 | 0.974665625 | 0.991780032 | 96730.69327 |
| 31 | 100067.5555 | 0.973832291 | 0.991066599 | 96578.46568 |
| 32 | 100067.5555 | 0.972999669 | 0.990291549 | 96420.4283 |
| 33 | 100067.5555 | 0.97216776 | 0.989449615 | 96256.08394 |
| 34 | 100067.5555 | 0.971336561 | 0.988535091 | 96084.8944 |
| 35 | 100067.5555 | 0.970506074 | 0.987541795 | 95906.27717 |
| 36 | 100067.5555 | 0.969676296 | 0.986463032 | 95719.60206 |
| 37 | 100067.5555 | 0.968847228 | 0.985291556 | 95524.1875 |

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| $x$ | K | $s^x$ | $g^{c(x)}$ | $l_x$ |
|---|---|---|---|---|
| 38 | 100067.555 | 0.968018869 | 0.9840195 | 95319.29 |
| 39 | 100067.555 | 0.967191217 | 0.9826384 | 95104.13 |
| 40 | 100067.555 | 0.966364274 | 0.9811391 | 94877.83 |
| 41 | 100067.555 | 0.965538038 | 0.9795117 | 94639.48 |
| 42 | 100067.555 | 0.964712508 | 0.9777455 | 94388.06 |
| 43 | 100067.555 | 0.963887683 | 0.9758290 | 94122.49 |
| 44 | 100067.555 | 0.963063564 | 0.9737495 | 93841.62 |
| 45 | 100067.555 | 0.96224015 | 0.9714939 | 93544.19 |
| 46 | 100067.555 | 0.96141744 | 0.9690475 | 93228.85 |
| 47 | 100067.555 | 0.960595433 | 0.966394 | 92894.15 |
| 48 | 100067.555 | 0.959774129 | 0.9635190 | 92538.5 |
| 49 | 100067.555 | 0.958953527 | 0.960402 | 92160.33 |
| 50 | 100067.555 | 0.958133627 | 0.9570253 | 91757.75 |
| 51 | 100067.555 | 0.957314427 | 0.9533673 | 91328.88 |
| 52 | 100067.555 | 0.956495928 | 0.9494063 | 90871.67 |
| 53 | 100067.555 | 0.955678129 | 0.9451186 | 90383.93 |
| 54 | 100067.555 | 0.95486103 | 0.9404790 | 89863.34 |
| 55 | 100067.555 | 0.954044628 | 0.9354608 | 89307.43 |
| 56 | 100067.555 | 0.953228925 | 0.930035 | 88713.56 |
| 57 | 100067.555 | 0.952413919 | 0.924172 | 88078.97 |
| 58 | 100067.555 | 0.95159961 | 0.9178410 | 87400.72 |
| 59 | 100067.555 | 0.950785998 | 0.9110064 | 86675.73 |
| 60 | 100067.555 | 0.949973081 | 0.9036336 | 85900.75 |
| 61 | 100067.555 | 0.949160859 | 0.8956857 | 85072.41 |
| 62 | 100067.555 | 0.948349331 | 0.8871242 | 84187.20 |
| 63 | 100067.555 | 0.947538497 | 0.8779091 | 83241.46 |
| 64 | 100067.555 | 0.946728357 | 0.8679991 | 82231.45 |
| 65 | 100067.555 | 0.945918909 | 0.8573521 | 81153.34 |
| 66 | 100067.555 | 0.945110153 | 0.8459249 | 80003.23 |
| 67 | 100067.555 | 0.944302089 | 0.8336742 | 78777.21 |
| 68 | 100067.555 | 0.943494716 | 0.8205567 | 77471.39 |
| 69 | 100067.555 | 0.942688032 | 0.806529 | 76081.94 |
| 70 | 100067.555 | 0.941882039 | 0.7915514 | 74605.17 |
| 71 | 100067.555 | 0.941076735 | 0.7755829 | 73037.61 |
| 72 | 100067.555 | 0.940272119 | 0.7585875 | 71376.05 |
| 73 | 100067.555 | 0.939468191 | 0.7405326 | 69617.68 |
| 74 | 100067.555 | 0.938664951 | 0.721390 | 67760.17 |
| 75 | 100067.555 | 0.937862397 | 0.7011412 | 65801.82 |
| 76 | 100067.555 | 0.93706053 | 0.6797705 | 63741.64 |
| 77 | 100067.555 | 0.936259348 | 0.6572750 | 61579.56 |
| 78 | 100067.555 | 0.935458851 | 0.6336621 | 59316.53 |
| 79 | 100067.555 | 0.934659039 | 0.608952 | 56954.70 |
| 80 | 100067.555 | 0.93385991 | 0.5831796 | 54497.59 |

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| $x$ | K | $s^x$ | $g^{c(x)}$ | $l_x$ |
|---|---|---|---|---|
| 81 | 100067.555 | 0.933061465 | 0.55639630 | 51950.2665 |
| 82 | 100067.555 | 0.932263702 | 0.5286716 | 49319.4335 |
| 83 | 100067.555 | 0.931466621 | 0.50009496 | 46613.6452 |
| 84 | 100067.555 | 0.930670222 | 0.4707766 | 43843.3786 |
| 85 | 100067.555 | 0.929874504 | 0.44084899 | 41021.1173 |
| 86 | 100067.555 | 0.929079466 | 0.41046667 | 38161.3782 |
| 87 | 100067.555 | 0.928285108 | 0.3798063 | 35280.6758 |
| 88 | 100067.555 | 0.927491429 | 0.3490656 | 32397.4103 |
| 89 | 100067.555 | 0.926698429 | 0.31846101 | 29531.6687 |
| 90 | 100067.555 | 0.925906106 | 0.28822464 | 26704.9245 |
| 91 | 100067.555 | 0.925114461 | 0.25860012 | 23939.6334 |
| 92 | 100067.555 | 0.924323493 | 0.22983693 | 212587.1937 |
| 93 | 100067.555 | 0.923533202 | 0.20218379 | 18684.9587 |
| 94 | 100067.555 | 0.922743586 | 0.17588105 | 16240.2756 |
| 95 | 100067.555 | 0.921954645 | 0.15115232 | 13944.9732 |
| 96 | 100067.555 | 0.921166378 | 0.12819573 | 11816.9378 |
| 97 | 100067.555 | 0.920378786 | 0.10717540 | 9870.860573 |
| 98 | 100067.555 | 0.919591867 | 0.08821363 | 8117.53386 |
| 99 | 100067.555 | 0.91880562 | 0.0713845 | 6563.28158 |
| 100 | 100067.555 | 0.918020046 | 0.05670973 | 5209.58433 |
| 101 | 100067.555 | 0.917235144 | 0.04415678 | 4052.95191 |
| 102 | 100067.555 | 0.916450913 | 0.0336406 | 3085.08042 |
| 103 | 100067.555 | 0.915667352 | 0.02502829 | 2293.30738 |
| 104 | 100067.555 | 0.914884461 | 0.01814685 | 1661.34898 |
| 105 | 100067.555 | 0.91410224 | 0.01279375 | 1170.27003 |
| 106 | 100067.555 | 0.913320687 | 0.00874903 | 799.607019 |
| 107 | 100067.555 | 0.912539803 | 0.005788066 | 528.540858 |
| 108 | 100067.555 | 0.911759586 | 0.0036937 | 337.005935 |
| 109 | 100067.555 | 0.910980036 | 0.00226666 | 206.628271 |
| 110 | 100067.555 | 0.910201153 | 0.00133297 | 121.409434 |
| 111 | 100067.555 | 0.909422936 | 0.00074843 | 68.1104362 |
| 112 | 100067.555 | 0.90864583 | 0.00039960 | 36.3346435 |
| 113 | 100067.555 | 0.907868497 | 0.00020200 | 18.3513857 |
| 114 | 100067.555 | 0.907092274 | 9.62E-05 | 8.73345584 |
| 115 | 100067.555 | 0.906316715 | 4.30E-05 | 3.89601569 |
| 116 | 100067.555 | 0.905541819 | 1.79E-05 | 1.62003437 |
| 117 | 100067.555 | 0.90476758 | 6.89E-06 | 0.62407233 |
| 118 | 100067.555 | 0.903994013 | 2.45E-06 | 0.22123760 |
| 119 | 100067.555 | 0.903221103 | 7.93E-07 | 0.07165559 |
| 120 | 100067.555 | 0.902448854 | 2.33E-07 | 0.02103720 |

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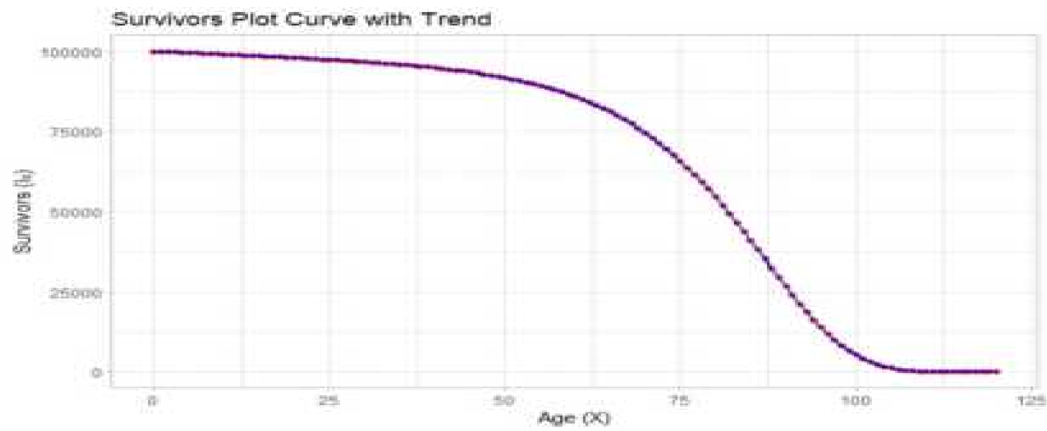


Figure 2: Graph of Survivors

The survivor table and curve jointly analyze the mortality trajectory modeled through the Gompertz-Makeham framework. The graph shows a real sigmoid (S-shaped) survivor curve, where mortality is low during early life and mid-adulthood but accelerates rapidly beyond age 60. This is consistent with the force of mortality data from the GM (1,2) table, where rates remain low during the first several decades of life and increase exponentially with age. The survivorship reduces slowly at first, with over 90,000 survivors still alive at age 50, but by age 80, the number living decreased drastically to about half. The steep decline aligns with recent observations of aging populations as validated by Souza (2022) on closed-form expressions to Gompertz and Gompertz-Makeham life expectancies for a person aged $x$ still been independently resolved. The force of mortality increases exponentially in late ages showing a sharp drop in survival, particularly beyond age 70 with a trend aligned with modern longevity data (Hunt & Blake, 2021). The consistency of the table data with the Human Mortality Database (2022) further supports the reliability of the modeled values. As the survivor count falls to zero by age 120, confirming Genesis 6:3

**Table 4: Curve of Death**

| | | | |
|---|---|---|---|
| 0 | 100000.3002 | 99908.94409 | 91.35611 |
| 1 | 99908.94409 | 99817.16134 | 91.78275 |
| 2 | 99817.16134 | 99724.90886 | 92.25248 |
| 3 | 99724.90886 | 99632.13983 | 92.76903 |
| 4 | 99632.13983 | 99538.80346 | 93.33637 |
| 5 | 99538.80346 | 99444.84455 | 93.95891 |
| 6 | 99444.84455 | 99350.2032 | 94.64135 |
| 7 | 99350.2032 | 99254.81436 | 95.38884 |
| 8 | 99254.81436 | 99158.60742 | 96.20694 |
| 9 | 99158.60742 | 99061.50568 | 97.10174 |
| 10 | 99061.50568 | 98963.42592 | 98.07976 |
| 11 | 98963.42592 | 98864.27776 | 99.14816 |
| 12 | 98864.27776 | 98763.96311 | 100.31465 |
| 13 | 98763.96311 | 98662.37546 | 101.58765 |
| 14 | 98662.37546 | 98559.39923 | 102.97623 |
| 15 | 98559.39923 | 98454.90895 | 104.49028 |
| 16 | 98454.90895 | 98348.76845 | 106.1405 |
| 17 | 98348.76845 | 98240.82995 | 107.9385 |
| 18 | 98240.82995 | 98130.93308 | 109.89687 |
| 19 | 98130.93308 | 98018.90382 | 112.02926 |
| 20 | 98018.90382 | 97904.55336 | 114.35046 |
| 21 | 97904.55336 | 97787.67688 | 116.87648 |
| 22 | 97787.67688 | 97668.05215 | 119.62473 |
| 23 | 97668.05215 | 97545.43817 | 122.61398 |
| 24 | 97545.43817 | 97419.57357 | 125.8646 |
| 25 | 97419.57357 | 97290.17489 | 129.39868 |
| 26 | 97290.17489 | 97156.93483 | 133.24006 |
| 27 | 97156.93483 | 97019.52028 | 137.41455 |
| 28 | 97019.52028 | 96877.57018 | 141.9501 |
| 29 | 96877.57018 | 96730.69327 | 146.87691 |
| 30 | 96730.69327 | 96578.46568 | 152.22759 |
| 31 | 96578.46568 | 96420.4283 | 158.03738 |
| 32 | 96420.4283 | 96256.08394 | 164.34436 |
| 33 | 96256.08394 | 96084.8944 | 171.18954 |
| 34 | 96084.8944 | 95906.27717 | 178.61723 |
| 35 | 95906.27717 | 95719.60206 | 186.67511 |
| 36 | 95719.60206 | 95524.1875 | 195.41456 |
| 37 | 95524.1875 | 95319.29667 | 204.89083 |

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| $x$ | $l_x$ | $l_{x+1}$ | $d_x$ |
|---|---|---|---|
| 38 | 95319.29667 | 95104.13333 | 215.16334 |
| 39 | 95104.13333 | 94877.83743 | 226.2959 |
| 40 | 94877.83743 | 94639.48051 | 238.35692 |
| 41 | 94639.48051 | 94388.06075 | 251.41976 |
| 42 | 94388.06075 | 94122.49791 | 265.56284 |
| 43 | 94122.49791 | 93841.62789 | 280.87002 |
| 44 | 93841.62789 | 93544.19718 | 297.43071 |
| 45 | 93544.19718 | 93228.85709 | 315.34009 |
| 46 | 93228.85709 | 92894.15773 | 334.69936 |
| 47 | 92894.15773 | 92538.542 | 355.61573 |
| 48 | 92538.542 | 92160.33936 | 378.20264 |
| 49 | 92160.33936 | 91757.75971 | 402.57965 |
| 50 | 91757.75971 | 91328.88726 | 428.87245 |
| 51 | 91328.88726 | 90871.67466 | 457.2126 |
| 52 | 90871.67466 | 90383.9373 | 487.73736 |
| 53 | 90383.9373 | 89863.34824 | 520.58906 |
| 54 | 89863.34824 | 89307.43364 | 555.9146 |
| 55 | 89307.43364 | 88713.56914 | 593.8645 |
| 56 | 88713.56914 | 88078.97726 | 634.59188 |
| 57 | 88078.97726 | 87400.72634 | 678.25092 |
| 58 | 87400.72634 | 86675.73115 | 724.99519 |
| 59 | 86675.73115 | 85900.75576 | 774.97539 |
| 60 | 85900.75576 | 85072.41896 | 828.3368 |
| 61 | 85072.41896 | 84187.20295 | 885.21601 |
| 62 | 84187.20295 | 83241.46578 | 945.73717 |
| 63 | 83241.46578 | 82231.45819 | 1010.00759 |
| 64 | 82231.45819 | 81153.3457 | 1078.11249 |
| 65 | 81153.3457 | 80003.23665 | 1150.10905 |
| 66 | 80003.23665 | 78777.21708 | 1226.01957 |
| 67 | 78777.21708 | 77471.39344 | 1305.82364 |
| 68 | 77471.39344 | 76081.94396 | 1389.44948 |
| 69 | 76081.94396 | 74605.1797 | 1476.76426 |
| 70 | 74605.1797 | 73037.61615 | 1567.56355 |
| 71 | 73037.61615 | 71376.05625 | 1661.5599 |
| 72 | 71376.05625 | 69617.68542 | 1758.37083 |
| 73 | 69617.68542 | 67760.1792 | 1857.50622 |
| 74 | 67760.1792 | 65801.82341 | 1958.35579 |
| 75 | 65801.82341 | 63741.64661 | 2060.1768 |
| 76 | 63741.64661 | 61579.56391 | 2162.0827 |
| 77 | 61579.56391 | 59316.53047 | 2263.03344 |
| 78 | 59316.53047 | 56954.70197 | 2361.8285 |
| 79 | 56954.70197 | 54497.59846 | 2457.10351 |
| 80 | 54497.59846 | 51950.26657 | 2547.33189 |
| 81 | 51950.26657 | 49319.43354 | 2630.83303 |
| 82 | 49319.43354 | 46613.64526 | 2705.78828 |
| 83 | 46613.64526 | 43843.37866 | 2770.2666 |
| 84 | 43843.37866 | 41021.11737 | 2822.26129 |
| 85 | 41021.11737 | 38161.37826 | 2859.73911 |
| 86 | 38161.37826 | 35280.67578 | 2880.70248 |
| 87 | 35280.67578 | 32397.41039 | 2883.26539 |
| 88 | 32397.41039 | 29531.66873 | 2865.74166 |
| 89 | 29531.66873 | 26704.92454 | 2826.74419 |
| 90 | 26704.92454 | 23939.63341 | 2765.29113 |
| 91 | 23939.63341 | 21258.71937 | 2680.91404 |
| 92 | 21258.71937 | 18684.95877 | 2573.7606 |
| 93 | 18684.95877 | 16240.27562 | 2444.68315 |
| 94 | 16240.27562 | 13944.97324 | 2295.30238 |
| 95 | 13944.97324 | 11816.93781 | 2128.03543 |
| 96 | 11816.93781 | 9870.860573 | 1946.077237 |
| 97 | 9870.860573 | 8117.533861 | 1753.326712 |
| 98 | 8117.533861 | 6563.28158 | 1554.252281 |
| 99 | 6563.28158 | 5209.58433 | 1353.69725 |
| 100 | 5209.58433 | 4052.951919 | 1156.632411 |
| 101 | 4052.951919 | 3085.080421 | 967.871498 |
| 102 | 3085.080421 | 2293.307388 | 791.773033 |
| 103 | 2293.307388 | 1661.348985 | 631.958403 |
| 104 | 1661.348985 | 1170.270035 | 491.07895 |
| 105 | 1170.270035 | 799.6070194 | 370.6630156 |
| 106 | 799.6070194 | 528.5408588 | 271.0661606 |
| 107 | 528.5408588 | 337.005935 | 191.5349238 |
| 108 | 337.005935 | 206.6282712 | 130.3776638 |
| 109 | 206.6282712 | 121.4094349 | 85.2188363 |
| 110 | 121.4094349 | 68.11043622 | 53.29899868 |
| 111 | 68.11043622 | 36.33464353 | 31.77579269 |
| 112 | 36.33464353 | 18.3513857 | 17.98325783 |
| 113 | 18.3513857 | 8.733455844 | 9.617929856 |
| 114 | 8.733455844 | 3.896015697 | 4.837440147 |
| 115 | 3.896015697 | 1.620034378 | 2.275981319 |
| 116 | 1.620034378 | 0.624072338 | 0.99596204 |
| 117 | 0.624072338 | 0.221237604 | 0.402834734 |
| 118 | 0.221237604 | 0.071655597 | 0.149582007 |
| 119 | 0.071655597 | 0.021037203 | 0.050618394 |
| 120 | 0.021037203 | 0 | 0.021037203 |

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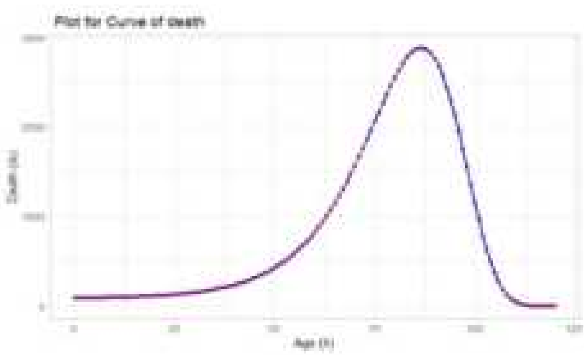


Figure 3: Graph or Curve of Death

The Curve of Death, which plots the number of deaths d$x$ against age ($x$), reveals a characteristic bell-shaped distribution observed in most mortality studies. This plot shows low death rates during childhood and early adulthood, followed by a gradual increase in mortality starting around accidental hump ages and climaxing sharply between ages 85 and 90. After the climax, the curve drops sharply due to a low number of individuals surviving to old age. This mortality pattern is compatible with findings from global demographic studies, which show that life expectancy has improved, but aging remains the dominant cause of death in high- and middle-income countries (United Nations, 2024). Mortality patterns also aid in the construction of life tables, which are crucial for assessing population health and financial liabilities linked to aging (OECD, 2023). Thus, the Curve of Death not only reveals old age death but also serves as a critical tool for health, economic, and social planning in the aging world.

**CONCLUSION**

In conclusion, this study successfully applied the GM (1,2) model to estimate key mortality indicators such as the force of mortality, survivorship curve, and curve of death. The analytical results indicated the model's position to reveal and forecast mortality trends using incomplete datasets, which is particularly valuable in contexts where perfect demographic data are unavailable or undependable. The derived survivorship and death curves aligned with recognized mortality trends over increased longevity and a numerous death at older ages. The results of all the parametric variables lies within interval of actuarial standard as validated by Bower's et al (1997).

The actuarial standard values of validities are 0.001< A< 0.003, 0.000001<B<0.003 and 0.008<C<1.12 which the results of this study falls within range i.e. 0.00100581246,0.00005629822$AB$==and C=1.08717487983. These findings support the applicability of the GM (1,2) model for life table construction and demographic analysis in both actuarial and public health. This unveiled that as age increases, the rate of deaths increases and exponentially increases as age goes older reaching omega ($w$) i.e. smallest age beyond which no life exists.

This conclusion is corresponded with prior studies that highlight the effectiveness of GM (1,2) model in demographic forecasting. For instance, Chakfa et al (2024) concludes that mortality increases with respect to age and as human grow older, the mortality rate becomes higher. Castellares, Patricio, Lemonte, & Queiroz. (2020) affirmed the predictive accuracy of GM-based model in estimating mortality and fertility rates in data-scarce environments, while Liu and Deng (2021) demonstrated the flexibility of GM (1,2) model in health-related projections. Moreover, this study can be validated by the earlier work of Liu, Zhu and Kai (2022). who emphasized the numerical use of mathematical modeling in aging population analysis. By applying the GM (1,2) model to mortality estimation, this research outcome contributes to the web of sciences in the growing body of proof supporting Gompertz-Makeham theory as a dependable tool for demographic prediction in actuarial science. Ultimately, it offers a better approach for policymakers, insurers, and researchers dealing with partial mortality data.

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